Unit 7 – Differential Equations

Modeling Situations with Differential Equations:

The modeling of situations with differential equations allows for scenarios such as the rate of change of velocity ($\frac{dv}{dt}$) with its equality of proportional (kx), inversely proportional (k/x), or squared (kx^2) with respect to x.

EX#1: The warming or cooling rate of a drink is proportional to the difference between the ambient temperature *Ta* and the current temperature *T* of the drink. Write an expression that models this relationship.

EX#2: The rate of change of the perceived stimulus *p* with respect to the measured intensity *s* of the stimulus is inversely proportional to the intensity of the stimulus. Write an expression that models this relationship.

EX#3: A radioactive material decays at a rate of change proportional to the current amount, *Q*, of the radioactive material. Write an expression that models this relationship.

Verifying Solutions for Differential Equations:

EX#1:

$$\frac{dy}{dx} = \frac{5y}{x}$$

Is $y = -2x^5$ a solution to the above equation?

Slope Fields:

A **slope field** is a collection of short line segments, whose ____________ match that of a solution of a first-order differential equation passing through the segment's midpoint. The pattern produced by the **slope field** aids in visualizing the shape of the curve of the solution.

EX#1:

In drawing the slope field for the differential equation $\frac{dy}{dx} = x + 2y - 2$, I would place short line segments at select points on the xy-plane.

Complete the sentences.

At the point $(-2, 0)$, I would draw a short segment of slope ______.

At the point $(0, 3)$, I would draw a short segment of slope ______.

At the point $(1, 1)$, I would draw a short segment of slope ______.

Unit 7 – Differential Equations

Finding Solutions Using Separation of Variables:

DIFFERENTIAL EQUATIONS (**Separating Variables**) (used when you are given the derivative and you need to find the original equation. We separate the x's and y's and take the integral).

EX#1: Find the general solution given $\frac{dy}{dx} = \frac{x^2}{y}$

EX#2: Find the particular solution $y = f(x)$ for **EX#1** given $(3, -5)$

EX#3: Find the particular solution $y = f(x)$ given $\frac{dy}{dx} = 6xy$ and $(0, 5)$

Exponential Models with Differential Equations:

If the rate of growth of something is proportional to itself $(y' = ky)$, then it is the growth formula.

Proof: $y' = ky \Rightarrow$

Sample AP Problems:

2013 AP Practice Exam Multiple Choice

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit (°F). If the initial temperature of the tea, at time $t = 0$ minutes, is 200°F and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) 175°F (B) 130°F (C) 95°F (D) 70°F (E) 45°F

Unit 7 – Differential Equations

90. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k ?

(A) 0.058 (B) 0.061 (C) 0.167 (D) 0.693 (E) 8.318

<u>2014 AP Practice Exam Multiple Choice</u>

18. A student attempted to solve the differential equation $\frac{dy}{dx} = xy$ with initial condition $y = 2$ when $x = 0$. In which step, if any, does an error first appear?

Step 1: $\int \frac{1}{y}\, dy = \int x\, dx$

Step 2: $\ln|y| = \frac{x^2}{2} + C$

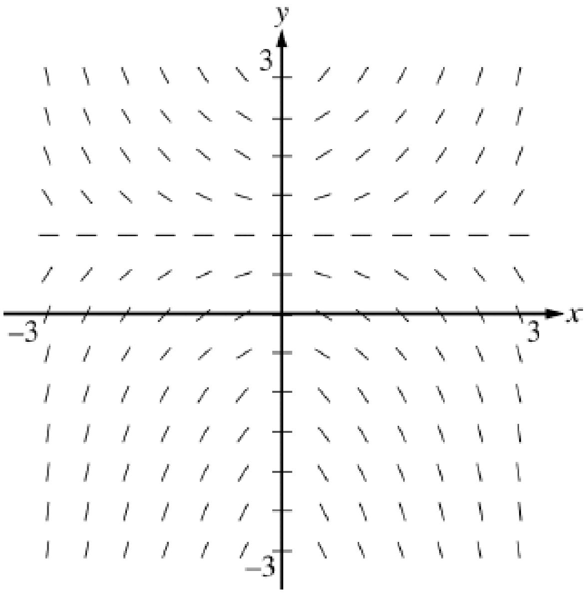
Step 3: $|y| = e^{x^2/2} + C$

Step 4: Since $y = 2$ when $x = 0$, $2 = e^0 + C$.

Step 5: $y = e^{x^2/2} + 1$

(A) Step 2
(B) Step 3
(C) Step 4
(D) Step 5
(E) There is no error in the solution.

Unit 7 – Differential Equations



28. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy - x$

(B) $\frac{dy}{dx} = xy + x$

(C) $\frac{dy}{dx} = y - x^2$

(D) $\frac{dy}{dx} = (y-1)x^2$

(E) $\frac{dy}{dx} = (y-1)^3$

81. At time $t = 0$ years, a forest preserve has a population of 1500 deer. If the rate of growth of the population is modeled by $R(t) = 2000e^{0.23t}$ deer per year, what is the population at time $t = 3$?

(A) 3987 (B) 5487 (C) 8641 (D) 10,141 (E) 12,628