Unit 5 – Analytical Applications of Differentiation

Properties of First Derivatives:

Derivative is a **rate of change**; it finds the change in y over the change in x, $\frac{dy}{dx}$, which is slope.

1st derivative $\Rightarrow$ max. and min., increasing and decreasing, slope of the tangent line to the curve, and velocity.

2nd derivative $\Rightarrow$ inflection points, concavity, and acceleration.

Properties of First Derivative:

Increasing: slopes of tangent lines are ________________ $f'(x) >$ ____

Decreasing: slopes of tangent lines are ________________ $f'(x) <$ ____

Maximum Point: Set $f'(x) =$ ____ and it is where the slopes turn from _______________ to _____________

Minimum Point: Set $f'(x) =$ ____ and it is where the slopes turn from _______________ to _____________

Properties of Second Derivative:

Concave Up: slopes of tangent lines are ________________ $f''(x) >$ ____

Concave Down: slopes of tangent lines are ________________ $f''(x) <$ ____

Inflection Points: Set $f''(x) =$ ____ and it is where the points on the graph switch ________________

EX#1: (a) Find the maximum point, the minimum point, the intervals of increasing, and the intervals of decreasing for the following function:

$$y = 2x^3 - 3x^2 - 36x + 2$$

(b) Find the inflection point, and the intervals of concavity for the function in EX#1.

Optimization Problems:

1) Draw and label a picture.
2) Write equations that fit the scenario.
3) Combine equations into one equation.
4) Take the derivative and set it equal to 0.
5) Solve for the variable.

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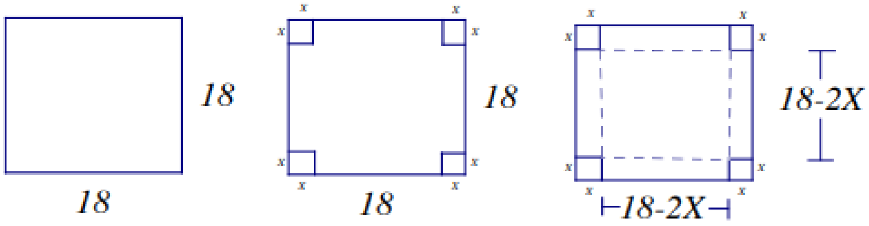
EX#1: An open box of maximum volume is to be made from a square piece of material, 18 inches on a side, by cutting equal squares from the corners and turning up the sides. How much should you cut off from the corners? What is the maximum volume of your box?

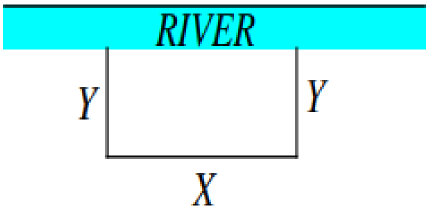


$V = (18-2x)^2 \cdot x$

$V = 4x^3 - 72x^2 + 324x$

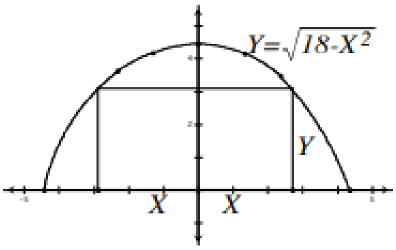
EX#2: A farmer plans to fence a rectangular pasture adjacent to a river. The farmer has 84 feet of fence in which to enclose the pasture. What dimensions should be used so that the enclosed area will be a maximum? What is the maximum Area?

$P = 2y + x \qquad A = x \cdot y$



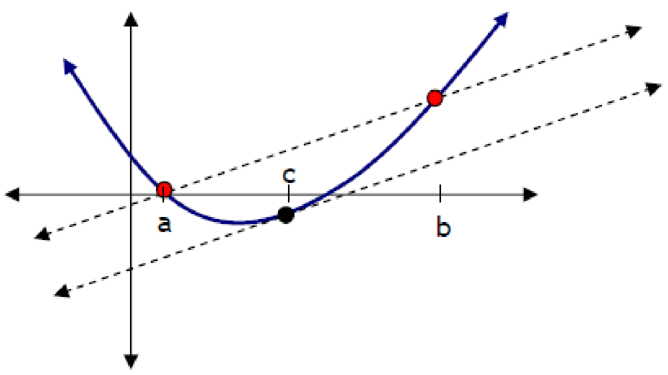
EX#3: A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{18 - x^2}$. What length and width should the rectangle have so that its area is a maximum?

$y = \sqrt{18 - x^2} \qquad A = 2 \cdot x \cdot y$



Mean Value Theorem of Derivatives:

$f'(\ ) = \dfrac{f(b) - f(a)}{b - a}$



The slope of the tangent at value c ______________ the slope of the secant through a and b.

EX#1: For what value c, such that $0 \le c \le 3$, is the instantaneous rate of change for $f(x) = x^2 - 2x$ equal to the average rate of change over the interval [0, 3]?

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Sample AP Problems:

2013 AP Practice Exam Multiple Choice

5. If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

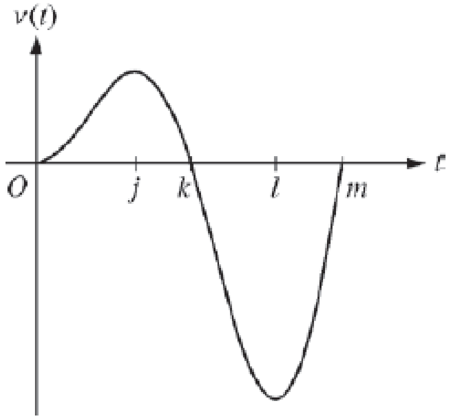
(A) $(-\infty, -10)$ and $(7, \infty)$

(B) $(-\infty, -7)$ and $(10, \infty)$

(C) $(-\infty, 10)$

(D) $(-10, 7)$

(E) $(-7, 10)$



8. A particle moves along a straight line. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 \le t \le m$, where j, k, l, and m are constants. The graph intersects the horizontal axis at $t = 0$, $t = k$, and $t = m$ and has horizontal tangents at $t = j$ and $t = l$. For what values of t is the speed of the particle decreasing?

(A) $j \le t \le l$

(B) $k \le t \le m$

(C) $j \le t \le k$ and $l \le t \le m$

(D) $0 \le t \le j$ and $k \le t \le l$

(E) $0 \le t \le j$ and $l \le t \le m$

13. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \le 3$ for all x. Of the following, which is not a possible value for $f(2)$?

(A) -10 (B) -5 (C) 0 (D) 1 (E) 2

24. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\frac{3}{4}$ (C) 0 (D) 2 (E) 6

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28. The function f is defined by $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$. What is the x-coordinate of the point of inflection where the graph of f changes from concave down to concave up?

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$ (E) $\frac{9\pi}{4}$

82. The derivative of the function f is given by $f'(x) = x^3 - 4\sin(x^2) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

(A) -1.970 and 0

(B) -1.467 and 1.075

(C) -0.475, 0.542, and 1.396

(D) -0.475 and 1.396 only

(E) 0.542 only

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
–1	4
0	3
1	1

(B)

x	$f(x)$
–1	4
0	4
1	4

(C)

x	$f(x)$
–1	4
0	5
1	6

(D)

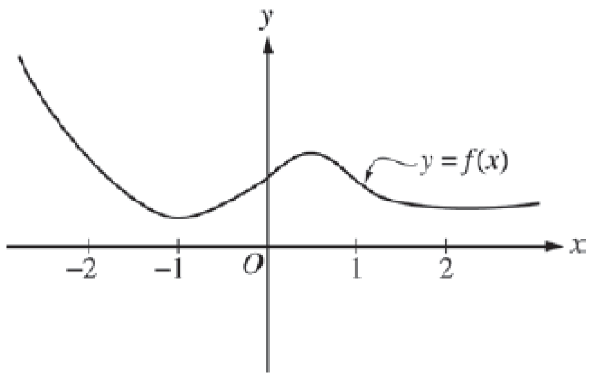
x	$f(x)$
–1	4
0	5
1	7

(E)

x	$f(x)$
–1	4
0	6
1	7

87. Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two (B) Three (C) Four (D) Five (E) Six



88. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-1) < f'(1) < f'(0)$

(B) $f'(-1) < f'(0) < f'(1)$

(C) $f'(0) < f'(-1) < f'(1)$

(D) $f'(1) < f'(-1) < f'(0)$

(E) $f'(1) < f'(0) < f'(-1)$

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92. The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) f is not continuous on $[a, b]$.

(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.

2014 AP Practice Exam Multiple Choice

9. The function f has a first derivative given by $f'(x) = x(x-3)^2(x+1)$. At what values of x does f have a relative maximum?

(A) -1 only (B) 0 only (C) -1 and 0 only (D) -1 and 3 only (E) -1, 0, and 3

15. The function $y = g(x)$ is differentiable and increasing for all real numbers. On what intervals is the function $y = g\left(x^3 - 6x^2\right)$ increasing?

(A) $(-\infty, 0]$ and $[4, \infty)$ only

(B) $[0, 4]$ only

(C) $[2, \infty)$ only

(D) $[6, \infty)$ only

(E) $(-\infty, \infty)$

19. For what values of x does the graph of $y = 3x^5 + 10x^4$ have a point of inflection?

(A) $x = -\frac{8}{3}$ only

(B) $x = -2$ only

(C) $x = 0$ only

(D) $x = 0$ and $x = -\frac{8}{3}$

(E) $x = 0$ and $x = -2$

22. Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

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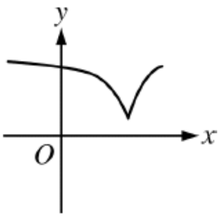
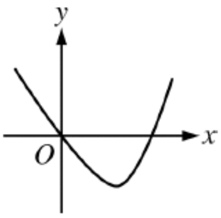
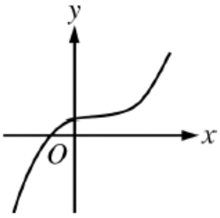
78. The function f is differentiable and increasing for all real numbers x, and the graph of f has exactly one point of inflection. Of the following, which could be the graph of f', the derivative of f?

(A)


(B)


(C)


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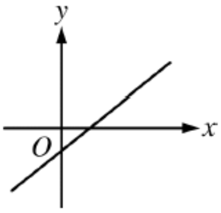
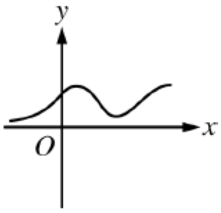


(E)


x	$f(x)$
1	2.4
3	3.6
5	5.4

80. The table above gives selected values of a function f. The function is twice differentiable with $f''(x) > 0$. Which of the following could be the value of $f'(3)$?

(A) 0.6 (B) 0.7 (C) 0.9 (D) 1.2 (E) 1.5

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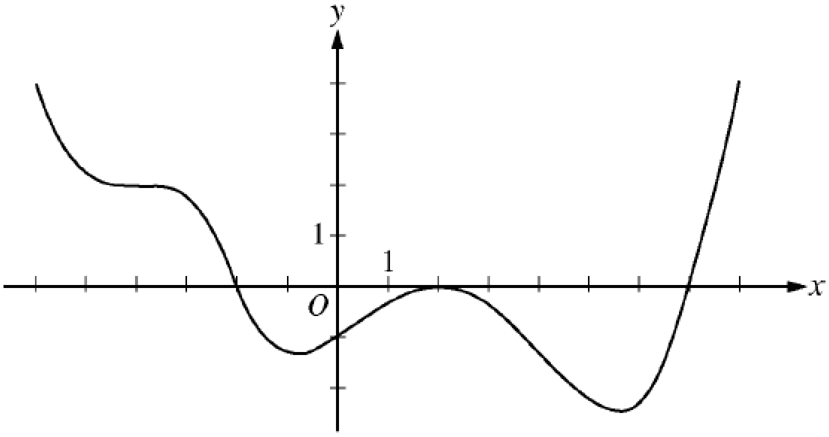

Graph of f'

82. The figure above shows the graph of f', the derivative of function f, for $-6 < x < 8$. Of the following, which best describes the graph of f on the same interval?

(A) 1 relative minimum, 1 relative maximum, and 3 points of inflection

(B) 1 relative minimum, 1 relative maximum, and 4 points of inflection

(C) 2 relative minima, 1 relative maximum, and 2 points of inflection

(D) 2 relative minima, 1 relative maximum, and 4 points of inflection

(E) 2 relative minima, 2 relative maxima, and 3 points of inflection

91. Let F be a function defined for all real numbers x such that $F'(x) > 0$ and $F''(x) > 0$. Which of the following could be a table of values for F ?

(A)

x	$F(x)$
1	–3
2	–4
3	–6
4	–9

(B)

x	$F(x)$
1	–3
2	–1
3	3
4	19

(C)

x	$F(x)$
1	–3
2	0
3	3
4	6

(D)

x	$F(x)$
1	–3
2	5
3	11
4	13

(E)

x	$F(x)$
1	–3
2	–4
3	–3
4	0