Unit 4 - Contextual Applications of Differentiation

*Rectilinear Motion (Position, Velocity, Acceleration Problems)

-We designate position as $x(t), y(t), or\ s(t)$.

-The derivative of position, $x'(t) = v(t) \Rightarrow$ velocity.

-The derivative of velocity, $v'(t) = a(t) \Rightarrow$ acceleration.

-We often talk about position, velocity, and acceleration when we're discussing particles moving along the x-axis or y-axis.

-A particle is at rest or is changing direction when $v(t) = 0$.

-A particle is moving to the right or up when $v(t) > 0$ and to the left or down when $v(t) < 0$.

Related Rates

We take derivatives with respect to t which allows us to find velocity. Here is how you take a derivative with respect to t:

derivative of x is $\frac{dx}{dt}$, derivative of y^2 is $2y\frac{dy}{dt}$, derivative of r^3 is $3r^2\frac{dr}{dt}$, derivative of t^2 is $2t\frac{dt}{dt} = 2t$

V means volume ; $\frac{dV}{dt}$ means rate of change of volume (how fast the volume is changing)

r means radius ; $\frac{dr}{dt}$ means rate of change of radius (how fast the radius is changing)

$\frac{dx}{dt}$ is how fast x is changing; $\frac{dy}{dt}$ is how fast y is changing

Volume of a sphere	Surface Area of a sphere	Area of a circle	Circumference of a circle
$V = \frac{4}{3}\pi r^3$	$A = 4\pi r^2$	$A = \pi r^2$	$C = 2\pi r$
$\frac{dV}{dt} =$	$\frac{dA}{dt} =$	$\frac{dA}{dt} =$	$\frac{dC}{dt} =$

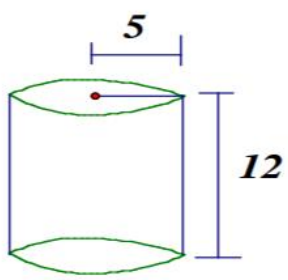
EX #1: **The radius of a spherical balloon is increasing at the rate of 4 *ft / min*. How fast is the surface area of the balloon changing when the radius is 3 *ft.*?**

$A = 4\pi r^2 \quad \Rightarrow \quad \frac{dA}{dt} =$

EX #2: **Water is poured into a cylinder with radius 5 at the rate of 10 in^3 / s. How fast is the height of the water changing when the height is 6 *in*?**

$V = \pi r^2 h$

$\frac{dV}{dt} =$



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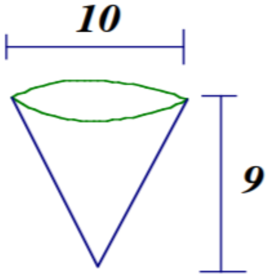
EX #3: **Water is leaking out of a cone with diameter 10 inches and height 9 inches at the rate of 7 in^3/s. How fast is the height of the water changing when the height is 5 in?**

$\frac{r}{h} =$ $\quad r =$

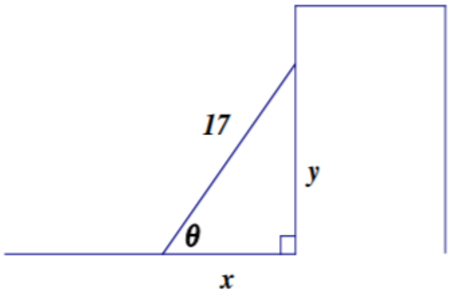
$V = \frac{1}{3}\pi r^2 h$



EX #4: **A 17 foot ladder is leaning against the wall of a house. The base of the ladder is pulled away at 3 ft. per second.**

a) How fast is the ladder sliding down the wall when the base of the ladder is 8 ft. from the wall?

$x^2 + y^2 = 17^2 \Rightarrow y =$ when $x = 8$



b) How fast is the area of the triangle formed changing at this time?

$A = \frac{1}{2} x \cdot y$

c) How fast is the angle between the bottom of the ladder and the floor changing at this time?

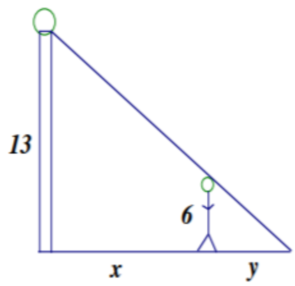
$\sin\theta = \frac{y}{17}$

EX #5: **A person 6 ft. tall walks directly away from a streetlight that is 13 feet above the ground. The person is walking away from the light at a constant rate of 2 feet per second.**

a) At what rate, in feet per second, is the length of the shadow changing?

$\frac{dx}{dt} =$ $\quad\quad \frac{dy}{dt} = ?$ (speed of the length of shadow)

Use similar triangles: $\frac{13}{x+y} = \frac{6}{y} \Rightarrow$



b) At what rate, in feet per second, is the tip of the shadow changing?

Tip of shadow is $x + y$, so speed of tip is

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Local Linearity and Linearization

** This helps us _______________________ a function using the tangent line to the curve.

If *f(a)* = a number you know and *x* is very close to *a*, then you do approximate *f(x)* by:

f(x) =

EX#1 - What is the approximate value of $f(x) = \sqrt[3]{x}$ when $x = 8.2$?

f(8) = *f'(x)* = *f'(8)* = *(x - a)* =

f(8.2) =

L'Hospital's Rule for Determining Limits of Indeterminate Forms

If you are taking the limit of a function that is indeterminate and equals, 0/0 or infinity/infinity, simply keep taking the derivative of the top and bottom functions until you get a limit.

$$\text{If } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\infty}{\infty}\text{, then } \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

EX#1 -

$$\lim_{x\to 2}\frac{x^2-4}{x-2}$$

EX#2 -

$$\lim_{x\to 1}\frac{\ln x}{x-1}$$

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Sample AP Problems:

2013 AP Practice Exam Multiple Choice

12. For which of the following does $\lim_{x\to\infty} f(x) = 0$?

I. $f(x) = \dfrac{\ln x}{x^{99}}$

II. $f(x) = \dfrac{e^x}{\ln x}$

III. $f(x) = \dfrac{x^{99}}{e^x}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?
(The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

85. A particle moves along the x-axis so that at time $t \ge 0$ its position is given by $x(t) = \cos\sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

(A) -1 (B) -0.624 (C) -0.318 (D) 0 (E) 0.065

2014 AP Practice Exam Multiple Choice

7. $\lim_{x\to\infty} \dfrac{x^3}{e^{3x}}$ is

(A) 0 (B) $\frac{2}{9}$ (C) $\frac{2}{3}$ (D) 1 (E) infinite

21. Functions w, x, and y are differentiable with respect to time and are related by the equation $w = x^2y$. If x is decreasing at a constant rate of 1 unit per minute and y is increasing at a constant rate of 4 units per minute, at what rate is w changing with respect to time when $x = 6$ and $y = 20$?

(A) -384 (B) -240 (C) -96 (D) 276 (E) 384

27. A particle moves on the x-axis so that at any time t, $0 \le t \le 1$, its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

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90. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

(A) $(0, 0.177)$ and $(1.256, \infty)$

(B) $(0, 1.256)$ only

(C) $(0, 2.057)$ only

(D) $(0.177, 1.256)$ only

(E) $(0.177, 1.256)$ and $(2.057, \infty)$

19. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \dfrac{\pi}{2}$?

(A) -2 (B) 1 (C) $\pi - 2$ (D) $4 - \pi$