Unit 1 - Limits and Continuity

** When evaluating limits, we are checking around the point that we are ________________, NOT the point.

** Every time we find a limit, we need to check from the ________ and the _________ hand side.

** Breaking Points are points on the graph that are ___________ or where the graph is __________ into pieces.

Breaking Points:

1) ____________________ (when the denominator equals 0)

2) ____________________ (when a fractional power equals 0)

3) ____________________ (when the numerator and denominator equals 0)

4) ____________________ Functions (the # where the graph is split)

** If the left and right hand limits DISAGREE, then the limit ____________________________

** If the left and right hand limits AGREE, then the limit ________________________

** Even if you can plug in the value, the limit might not exist at that point. It might not exist from if the left hand and right hand sides will not agree.

For example: $f(x)=\begin{cases} 3 \text{ for } x \geq 1 \\ 1 \text{ for } x < 1 \end{cases}$ $\lim_{x\to 1} f(x)=$ because $\lim_{x\to 1^+} f(x)=$ $\lim_{x\to 1^-} f(x)=$

Note: In general when doing limits, $\frac{\#}{x\to 0}=$ $\frac{-\#}{x\to 0}=$ $\frac{\#}{x\to \infty}=$

Limits at Non-Breaking Points:

EX#1: $\lim_{x\to 1} x^3+x-5=$ **EX#2:** $\lim_{x\to 2} \sqrt{x+7}=$ **EX#3:** $\lim_{x\to 1} \frac{2x-1}{x+1}=$

Holes in the Graph:

EX#1: $\lim_{x\to 2} \frac{x^2+3x-10}{x-2}=$

Radicals: If a # makes a radical negative, the limit will not exist at that number.

EX #1: $\lim_{x\to 3^-} \sqrt{3-x}=$

EX #2: $\lim_{x\to 5^+} \sqrt{5-x}=$

EX #3: $\lim_{x\to -2} \sqrt{x+2}=$

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Asymptotes: When a point is undefined, we have to check a point that is close on the side we are approaching.

EX #1: $\lim_{x\to 5^-} \frac{3}{5-x} =$

EX #2: $\lim_{x\to -8} \frac{7}{x+8} =$

EX #3: $\lim_{x\to -3^+} \frac{3x}{x+3} =$

EX #4: $\lim_{x\to 9} \frac{10}{(x-9)^2} =$

Trigonometric Functions:

FACTS: $\lim_{x\to 0} \frac{\sin x}{x} =$ $\lim_{x\to 0} \frac{1-\cos x}{x} =$ $\lim_{x\to 0} \frac{\tan x}{x} =$

$\lim_{x\to 0} \frac{\sin ax}{bx} =$ $\lim_{x\to 0} \frac{1-\cos ax}{bx} =$ $\lim_{x\to 0} \frac{\tan ax}{bx} =$

EX#1: $\lim_{x\to 0} \frac{\sin x \tan x}{x^2} =$

EX#2: $\lim_{x\to 0} \frac{3\sin 3x}{8x} =$

EX#3: $\lim_{x\to 0} \frac{6\sin x \cos x}{5x} =$

EX#4: $\lim_{x\to \pi/2} \frac{5\tan 3x}{x} =$

Piece-Wise Functions:

$$f(x) = \begin{cases} 3-x & x < -3 \\ 2x+1 & -3 \le x < 4 \\ 9 & x \ge 4 \end{cases}$$

EX #1: $\lim_{x\to -3^+} f(x) =$

EX #2: $\lim_{x\to 4^+} f(x) =$

EX #3: $\lim_{x\to -3^-} f(x) =$

EX #4: $\lim_{x\to 4^-} f(x) =$

EX #5: $\lim_{x\to -3} f(x) =$

EX #6: $\lim_{x\to 4} f(x) =$

EX #7: $\lim_{x\to 7^+} f(x) =$

EX #8: $\lim_{x\to -5} f(x) =$

EX #9: $\lim_{x\to 2} f(x) =$

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Limits That Approach Infinity:

Check the powers of the numerator and denominator.

Check the powers of the numerator and denominator.

1) If the denominator (bottom) is a bigger power the limit =
2) If the numerator (top) is a bigger power the limit =
3) If powers are the same the limit =

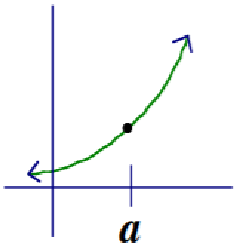
EX#1: $\lim\limits_{x\to\infty} \dfrac{3-5x^2}{13x^2+1} =$ **EX#2:** $\lim\limits_{x\to\infty} \dfrac{9-x^3}{x^2} =$ **EX#3:** $\lim\limits_{x\to\infty} \dfrac{1}{6-x} =$ **EX#4:** $\lim\limits_{x\to\infty} \dfrac{7-x}{x-7} =$

EX#5: $\lim\limits_{x\to-\infty} \dfrac{2x-5}{9x+1} =$ **EX#6:** $\lim\limits_{x\to-\infty} \dfrac{2x^5+3}{7x^2-5} =$ **EX#7:** $\lim\limits_{x\to-\infty} \dfrac{5-x}{3x^2+1} =$ **EX#8:** $\lim\limits_{x\to\infty} 3 =$

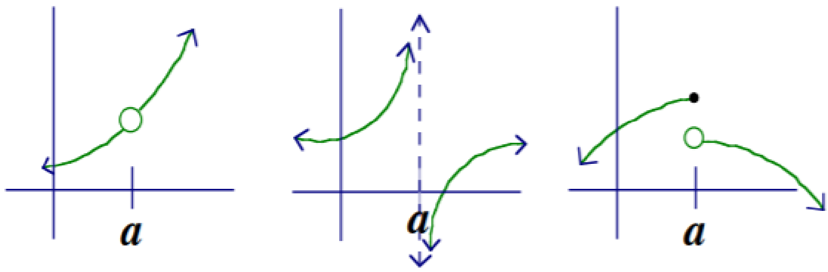
Continuity

** Continuous functions have ______ breaks in them.

** Discontinuous functions have _______ in them (Asymptotes or Holes)



Continuous at a



Discontinuous at a

** __________________ Discontinuity is when there is a hole

** __________________ Discontinuity is when there is an asymptote or a break

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Sample AP Problems

2013 AP Practice Exam Multiple Choice

1. $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$ is

(A) $-\dfrac{1}{4}$ (B) 0 (C) 1 (D) $\dfrac{5}{4}$ (E) nonexistent

9. Let f be the function given by $f(x) = \dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

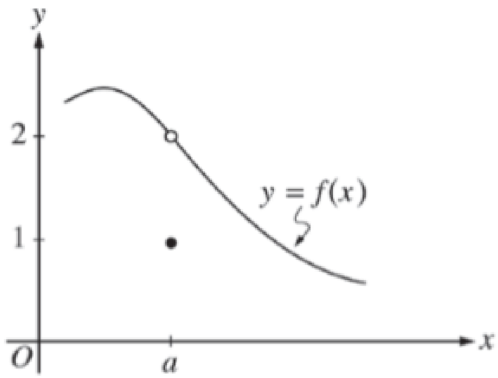
(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only



84. The graph of a function f is shown in the figure above. Which of the following statements is true?

(A) $f(a) = 2$

(B) f is continuous at $x = a$.

(C) $\lim_{x \to a} f(x) = 1$

(D) $\lim_{x \to a} f(x) = 2$

(E) $\lim_{x \to a} f(x)$ does not exist.

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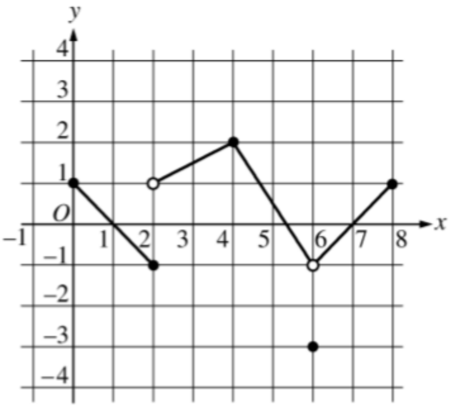
2014 AP Practice Exam Multiple Choice



5. The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

III. $\lim_{x \to 6} f(x) = f(6)$

(A) II only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

$$f(x) = \begin{cases} \dfrac{x^2 - 7x + 10}{b(x-2)} & \text{for } x \neq 2 \\ b & \text{for } x = 2 \end{cases}$$

10. Let f be the function defined above. For what value of b is f continuous at $x = 2$?

(A) -3 (B) $\sqrt{2}$ (C) 3 (D) 5 (E) There is no such value of b.

16. $\lim_{x \to 3^-} \dfrac{|x-3|}{x-3}$ is

(A) -3 (B) -1 (C) 1 (D) 3 (E) nonexistent

24. Let f be the function defined by $f(x) = \dfrac{(3x+8)(5-4x)}{(2x+1)^2}$. Which of the following is a horizontal asymptote to the graph of f?

(A) $y = -6$
(B) $y = -3$
(C) $y = -\dfrac{1}{2}$
(D) $y = 0$
(E) $y = \dfrac{3}{2}$