

## 2.11 Implicit Differentiation

FIND  $\frac{dy}{dx}$  FOR EACH OF THE FOLLOWING.

506.  $x^2 - y^2 = 5$

509.  $x = \tan y$

507.  $1 - xy = x - y$

510.  $x^3 - xy + y^3 = 1$

508.  $y^2 = x^3$

511.  $9x^2 + 25y^2 = 225$

512. Find the equation of both the tangent and normal lines to the curve  $x^2 \cos^2 y - \sin y = 0$  at the point  $(0, \pi)$ .

513. Find the equation of both the tangent and normal lines to the curve  $y^2(2 - x) = x^3$  at the point  $(1, 1)$ .

FIND  $\frac{d^2y}{dx^2}$  IN TERMS OF  $x$  AND  $y$  FOR THE FOLLOWING THREE PROBLEMS.

514.  $xy + y^2 = 1$

515.  $y^2 = x^2 + 2x$

516.  $x^2 + xy = 5$

517. Find the equation of the tangent line to the curve  $(x^2 + y^2)^2 = 4x^2y$  at the point  $(1, 1)$ .

518. Consider the curve defined by  $x^3 + y^3 - 9xy = 0$ .

- Find the equation of the tangent lines at the points  $(4, 2)$  and  $(2, 4)$ .
- At what points does the curve have a horizontal tangent?
- Find the coordinates of the point where the curve has a vertical tangent.

519. Find the two points where the curve  $x^2 + xy + y^2 = 7$  crosses the  $x$ -axis and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

520. The line that is normal to the curve  $x^2 + 2xy - 3y^2 = 0$  at the point  $(1, 1)$  intersects the curve at what other point?

521 (AP, 2000AB). Consider the curve given by  $xy^2 - x^3y = 6$ .

- Find  $\frac{dy}{dx}$ .
- Find all points on the curve whose  $x$ -coordinate is 1, and write an equation for the tangent line at each of these points.
- Find the  $x$ -coordinate of each point on the curve where the tangent is vertical.

## 2.17 Dérivé, Derivado, Ableitung, Derivative

639. Suppose that functions  $f(x)$  and  $g(x)$  and their first derivatives have the following values at  $x = 0$  and  $x = 1$ .

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	-3	$\frac{1}{2}$
1	3	5	$\frac{1}{2}$	-4

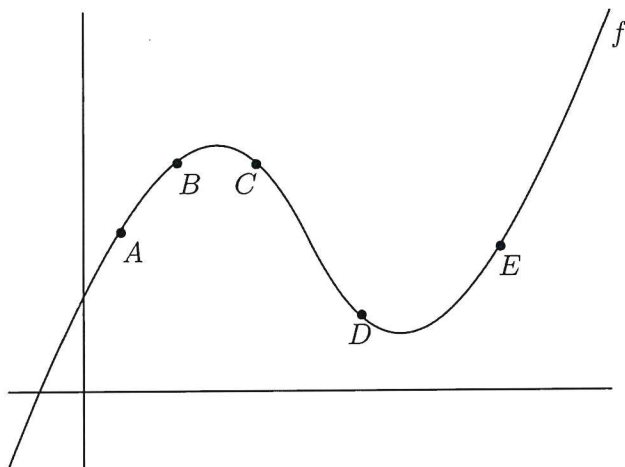
Find the first derivatives of the following combinations at the given value of  $x$ .

- |                                       |                                  |
|---------------------------------------|----------------------------------|
| a) $6f(x) - g(x)$ at $x = 1$          | d) $f(g(x))$ at $x = 0$          |
| b) $f(x)g^2(x)$ at $x = 0$            | e) $g(f(x))$ at $x = 0$          |
| c) $\frac{f(x)}{g(x) + 1}$ at $x = 1$ | f) $(x + f(x))^{3/2}$ at $x = 1$ |
|                                       | g) $f(x + g(x))$ at $x = 0$      |

640. If  $x^2 - y^2 = 1$ , find  $\frac{d^2y}{dx^2}$  at the point  $(2, \sqrt{3})$ .

641. For what values of  $a$  and  $b$  will  $f(x) = \begin{cases} ax & x < 2 \\ ax^2 - bx + 3 & x \geq 2 \end{cases}$  be differentiable for all values of  $x$ ?

642. Use the graph of  $f$  to answer the following.



- Between which two consecutive points is the average rate of change of the function greatest? Least?
- Is the average rate of change of the function between  $A$  and  $B$  greater than or less than the instantaneous rate of change at  $B$ ?
- Sketch a tangent line to the graph between the points  $D$  and  $E$  such that the slope of the tangent is the same as the average rate of the change of the function between  $D$  and  $E$ .
- Give a set of two points for which the average rate of change of the function is approximately equal to another set of two points.