Associating Limits with Graphical Behavior

You have in front of you three stacks of cards:

- One stack has cards showing eight different **graphs** of a function $f(x)$ with a specific region on the graph circled.
- Another stack has cards showing eight different **limit statements** in symbolic notation.
- The third stack has cards showing eight different **verbal descriptions** of graphical behavior.

Your task is to rearrange the cards into eight stacks, each containing a graph card, a limit card, and a verbal description card that match one another. Once you have finished this task, record your results in the following table, including the answer for each limit statement. If the limit is a finite number, write that number. If the function is unbounded in the positive or negative direction, write $+\infty$ or $-\infty$, respectively. Otherwise, write DNE for "does not exist." The first row of the table is filled out as an example.

Graphical Region	Limit Statement	Verbal Description	Answer
1	H	VI	$-\infty$
2			
3			
4			
5			
6			
7			
8			

Now share your findings with a partner and resolve any differences you have by discussing with your partner, and, if necessary, other members of the class.

The Connection Between Asymptotes and Limits

Part I: For each question, describe what the given information tells you about the graph of $y = f(x)$. Then, decide whether the given information allows you to identify a horizontal or vertical asymptote for the graph of $y = f(x)$. If there is sufficient information, state the equations of any asymptotes.

Example: $\lim_{x \to 2} f(x) = \infty$

- Explain what this tells you about the graph of $y = f(x)$

  As x gets closer to 2, both from the left and from the right, the y-coordinates are unbounded, getting larger and larger in the positive direction.

- Vertical asymptote(s)? ☐ No ☑ Yes, equation(s): ___**x = 2**_____
- Horizontal asymptote(s)? ☑ No ☐ Yes, equation(s): _____

1. $\lim_{x \to 3^-} f(x) = +\infty, \lim_{x \to 3^+} f(x) = -\infty$

- Explain what this tells you about the graph of $y = f(x)$

- Vertical asymptote(s)? ☐ No ☐ Yes, equation(s): _____
- Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s): _____

2. $\lim_{x \to +\infty} f(x) = 2$

- Explain what this tells you about the graph of $y = f(x)$

- Vertical asymptote(s)? ☐ No ☐ Yes, equation(s): _____
- Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s): _____

3. $\lim_{x \to -1^-} f(x) = 4, \lim_{x \to -1^+} f(x) = -\infty$

- Explain what this tells you about the graph of $y = f(x)$

- Vertical asymptote(s)? ☐ No ☐ Yes, equation(s): _____
- Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s): _____

4. $\lim_{x\to+\infty} f(x) = 2, \lim_{x\to-\infty} f(x) = 4$

- Explain what this tells you about the graph of $y = f(x)$

- Vertical asymptote(s)? ☐ No ☐ Yes, equation(s): _____
- Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s): _____

5. $\lim_{x\to4^-} f(x) = 2, \lim_{x\to4^+} f(x) = 3$

- Explain what this tells you about the graph of $y = f(x)$

- Vertical asymptote(s)? ☐ No ☐ Yes, equation(s): _____
- Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s): _____

6. $\lim_{x\to1} f(x) = +\infty, f(1) = 4$.

- Explain what this tells you about the graph of $y = f(x)$

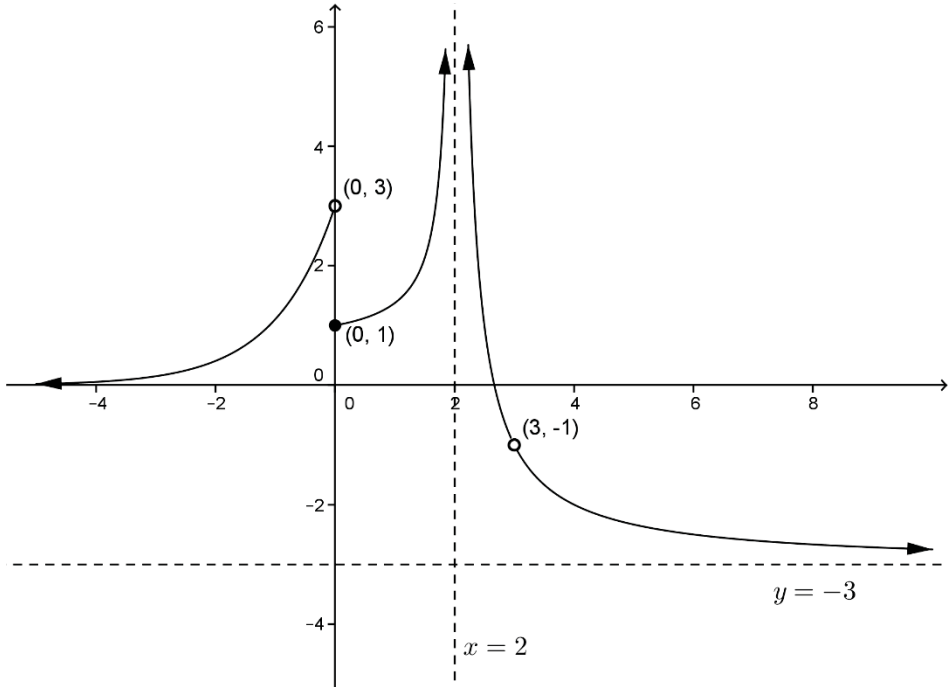
- Vertical asymptote(s)? ☐ No ☐ Yes, equation(s): _____
- Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s): _____

7. $\lim_{x\to2^+} f(x) = -\infty, \lim_{x\to3^-} f(x) = +\infty, \lim_{x\to+\infty} f(x) = +\infty, \lim_{x\to-\infty} f(x) = -\infty$

- Explain what this tells you about the graph of $y = f(x)$

- Vertical asymptote(s)? ☐ No ☐ Yes, equation(s): _____
- Horizontal asymptote(s)? ☐ No ☐ Yes, equation(s): _____

Part II: This part of the activity concentrates on writing correct notation for limit statements as well as making the connection between limits and graphical behavior. Shown below is the graph of a function $f(x)$. There are eleven limit statements, including one-sided and two-sided limits, based on the labeled points and lines on this graph. Write at least ten of these limit statements.



1._____ 6._____

2._____ 7._____

3._____ 8._____

4._____ 9._____

5._____ 10._____

11._____